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# Insights Into Oil Palm Yield Under Seasonal Rainfall

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## ABSTRACT

Young oil palms produce many small fruit bunches. With age, bunch number (BN) declines but single bunch weight (BW) increases more than the BN decline, raising its yield (FFB, or BN*BW). In a long-term trial in seasonal Southern Thailand, the age trend accounted for 81% of the variation in BN. With irrigation, BN increased 34%, and BW 5%, and the age trend accounted for 90% of BN variance. It was 98% for BW with/without irrigation. Besides age trends, the regular December–March dry season, despite irrigation, combined with intrinsic alternating sex cycles resulted in annual cycles in BN and BW. The BN cycle was more marked in younger palms whose rooting is shallower. The BW cycle persisted throughout, albeit at lower amplitude than BN. Female abortion after high production resulted in a BN semi-annual cycle, with peaks in Mar/Apr and Sep/Oct. A similar cycle for BW in older palms, with peaks in Dec–Feb and Jun–Aug, arose from fluctuating pollination. A three-year cycle in BN of unirrigated palms may be due to exhaustion/replenishment of carbohydrate reserves. Underripe harvesting, causing more yield in a month, and a dearth after, resulted in a 2-month cycle for BN.

*Keywords*: irrigation, oil palm, Thailand

## INTRODUCTION

The oil palm starts bearing (~2–3 years from field planting) by initially producing many small bunches, e.g., 15–20 bunches/year of average 3-5 kg weight. With age, the bunches increase in size but the number decreases, e.g., plateauing at 1–3 bunches of 40–60 kg at 20–30 years old, the end of its economic life. Average fruit size is largely unaffected. While this is the intrinsic nature of the palm, the enquiry is whither genetic and environmental effects. The question(s) was explored from 174 months (14.5 years) of yield (bunch number, and total and average single bunch weight) of individual progeny palms from first harvest at two years after planting. In this paper we report on aggregate behaviour. Progeny differences are discussed in a follow-up companion paper.

## MATERIALS AND METHODS

The 1999 trial comprised 13 DxP progenies grown with/ without irrigation at Univanich's TOPI estate (8°30' N, 98°50' E) in the Plai Phraya District of Krabi Province, South Thailand. The region typically

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experiences an annual 3–4-month dry season as in the rainfall records of the estate (Figure 1). Over a year, the rainfall is generally adequate for oil palm, if sub-optimal, but was higher in the last five years of the trial (Table1). In the irrigated block, each palm was drip irrigated at 300 L/day in the dry season (January to April) as per Palat *et al*. (2008).

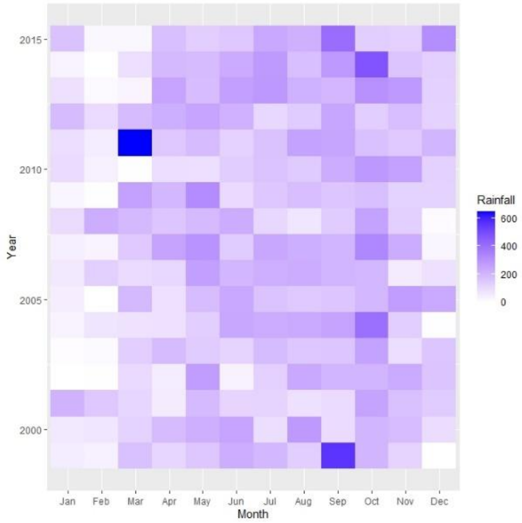


Figure 1 Monthly rainfall at trial location 1999–2015.

There were 234 experimental palms in the irrigated block and 468 in the unirrigated. Harvesting started in July 2001, 24 months after planting, and was done every 10–15 days, as per the industry norm. At each harvest, every bunch was weighed, and the data aggregated monthly. In brief, the bunch number (BN) and bunch weight (BW) were recorded from which could be derived the monthly FFB yield (BN* average BW). While yield in the first three months for the unirrigated area was recorded as zero from delayed bearing, if there was no yield from a particular block in subsequent months, i.e. BN =0, BW and FFB were recorded as 'NA'. There were seven such records in the irrigated data and four in the unirrigated. The subscripts 'irr' and 'nirr' denote the irrigated and unirrigated data.

The 174 months of continuous yield records of the two blocks were examined graphically and as time series data, i.e. a sequence of observations ordered over time, using R–a programming language for statistical computing, data visualisation and data analysis. Besides based on R, the following programme libraries were involved: tidyverse for data preparation, ggplot2 and lattice for data visualisation and astsa, rSSA and multitaper for time series analysis (TSA). Pretesting showed the time series data to be neither stationary nor ergodic (test results not shown) and this was considered throughout the analysis and interpretation, particularly the choice of programme libraries for TSA. There were a few outliers, but not removed as they actually occurred, i.e. not artifacts like recording errors (Sarkar 2008).

The primary analytical approach was to decompose and rebuild the time series by Singular Spectrum Analysis or SSA (Elsner and Tsonis 1996, Golyandina *et al.* 2001). SSA is data-adaptive and non-parametric, affording data representations that minimise global reconstruction error without the need for underlying statistical models. Unlike classical regression, efficient results are not premised on the stationarity, linearity, or normality of the data. Besides revealing trends, the method is superior to conventional techniques for detection of relatively weak and low frequency oscillations. As FFB, BN and BW trends with age are well known in oil palm, our primary interest was in deciphering seasonal effects with/without irrigation. Like PCA, but with wider applications, SSA decomposes data but into a number of time-shifted segments, sub series or components (Wickham 2016).

The correlation matrix estimated after embedding the single-dimension time series into its multi-dimension delayed coordinates or lagged vectors is eigen decomposed. In practice SSA consists of the following four steps: 1) segmentation of the time series into time-shifted or lagged components based on a defined time or window length. 2) their singular value eigen decomposition or SVD so that the first component accounts for the largest possi ble variance followed by succeeding lesser components, each separable from the pre-

Table 1 Annual rainfall (mm) and raindays at trial location.

| Year | 1999 | 2000 | 2001 | 2003 | 2004 | 2005 | 2007 | 2008 | 2009 | 2011 | 2012 | 2013 | 2014 | 2015 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rain-fall | 1995 | 1880 | 1716 | 1624 | 1915 | 1938 | 2198 | 1834 | 1842 | 2480 | 2114 | 2263 | 2265 | 2134 |
| Rain-days | 141 | 120 | 140 | 108 | 129 | 117 | 138 | 119 | 110 | 148 | 135 | 129 | 143 | 131 |

ceding and accounting for the highest of the remaining variance. SVD is widely used for reducing and denoising multi-dimensional data in machine learning, image and facial recognition from pixel capture, sound and voice recognition from digitised sound, natural language processing, etc. 3) a regrouping based on importance and separability of the components. High singular value single eigentriples mean trends, eigentriple pairs with close singular values denoting periodic components while a gradually decreasing sequence of singular values are typical of pure noise series. 4) a reconstruction of the time series from step (3) above. How well the reconstruction mirrors the original time series provides the confidence for forecasting. The slowly varying trend components may be intrinsic to the crop or stem from systemic changes in the growing environment. The trends may also change significantly at times. We used the R programme library 'astsa' to show these breakpoints. Oscillatory periodic or cyclical components, on the other hand, are often driven by seasonal environmental influences, endogenous cycles from hormonal or carbohydrate status and regular human interventions. Spectral profiling can aid discovery of periodic components and their importance. A high spectral density at a particular frequency implies periodicity at that frequency. However, with natural data, poor resolution and leakage can complicate their discovery. As a safeguard, we augmented the spectral analysis provided by Rssa with multitaper methods (MTM). Still, with natural data, removing trends and cyclicals will leave many non-independent components, each accounting for a small part of the variance. These are bulked and reported as residuals here but as 'noise' in other physical phenomena, particularly signal processing.

Variants of SSA have been developed and used for many types of temporal, spatial and sound series (Golyandina *et al* 2018 for examples). Given that BN and BW in oil palm show strong time trends, we opted to use a 2-step Sequential SSA variant; a first to isolate the trend and then repeat the SSA for the de-trended time series for cyclical components. The method is detailed below for $BN_{nirr}$ and $BW_{irr}$, two contrasting traits in different environments, followed by summaries for both traits in both environments (Rahim *et al.* 2014).

## RESULTS AND DISCUSSION

### Defining Trends

FFB and BW rose rapidly from start until about the 9$^{th}$ year after planting (Figure 2). Thereafter FFB plateaued while BW continued to increase, albeit at a slower pace. BN, on the other hand, was highest at start of bearing and then declined, the converse of the BW trend. It plateaued more gradually and, at any age, fluctuated more than BW (grey shaded area straddling the trend lines in Figure 2). Note that the low initial BN, of non-irrigated palms, especially in the first three months, is an artifact from not all the palms starting to yield together. Unsurprisingly, irrigated palms were more consistent in their start of bearing. The BN decline was immediate and continuous, contrary to Corley & Tinker (2016) who found the onset only after 6–10 years of bearing. However, the faster increase in BW contrived a rising FFB. Only after ~10 years of bearing, with BW ~20kg, did the decline in BN begin to offset the increasing BW, resulting in a plateauing FFB. There is debate whether FFB declines thereafter, albeit very gradually. A simple regression of FFB on age, suggests

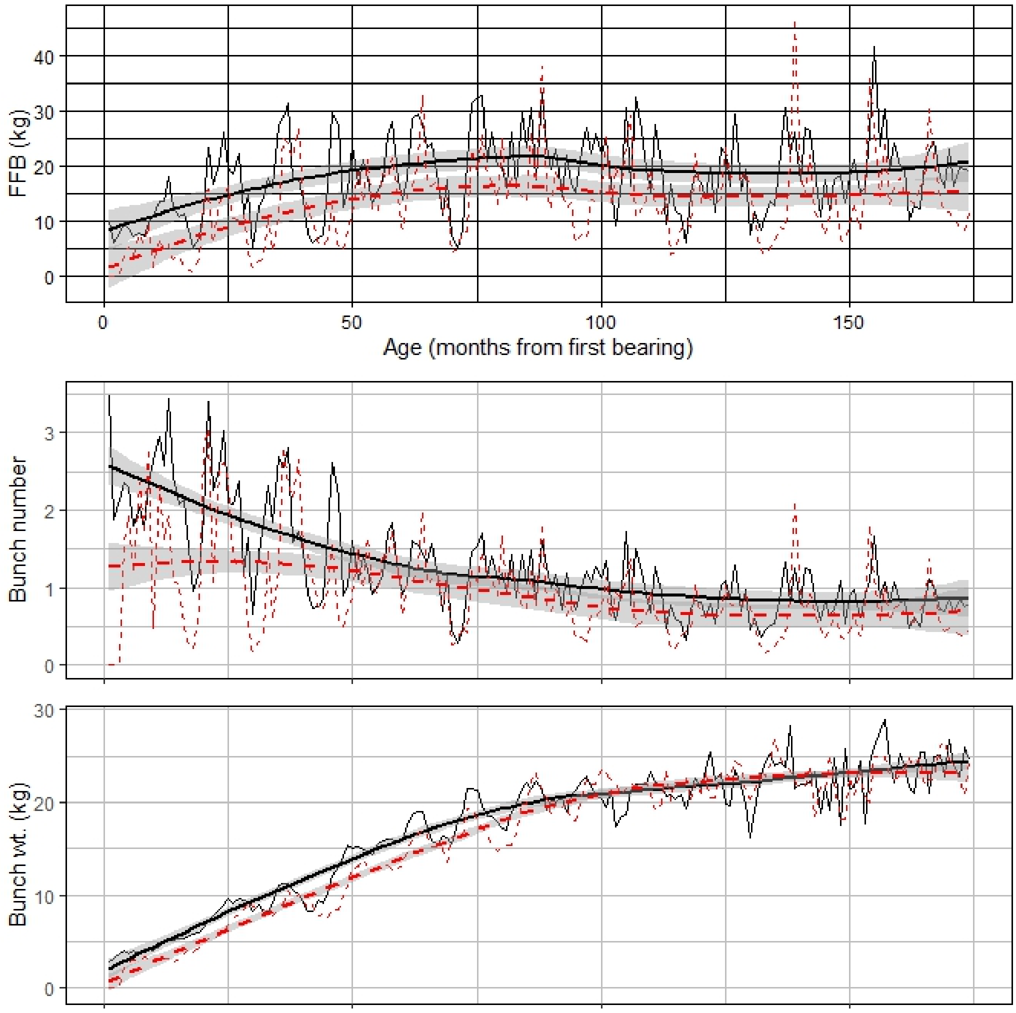


Figure 2 Trends in monthly FFB, BN and BW throughout trial for irrigated (solid line) and unirrigated (dashed line) palms. Straddling grey area is the 95% confidence interval.

that this was not so, until at least the end of the trial, although the areal yield may have done so from increasing dead palms and more missed bunches from the tallish palms.

**Irrigated and Unirrigated**

Throughout the trial the irrigated palms produced ~34% more bunches, which also averaged ~5% larger, resulting in a yield gain of ~38% (Figure 1). However, from the 8$^{th}$ year of bearing the higher yield only resulted from higher BN as both the irrigated and unirrigated BW had converged. This is probably due to the more robust root system in the older palms tapping into deeper soil water as against mainly the surface moisture previously. As the alleviation of water stress may have helped in the BW convergence, it is likely that better rainfall would be useful, and drought the converse. Interestingly, FFB fluctuated in tandem between the irrigated and unirrigated palms throughout the trial, i.e. irrigation did not dampen it. So other factors may be involved as well, such as the palm carbohydrate status.

**Breakpoints**

Analysis of the trends, using the R package astsa, (identifying structural breaks using piecewise AR models) revealed break points in both BN and BW (Figure 3). There was a structural change in BN after about the 4$^{th}$ year of bearing in irrigated palms and a year sooner in the un-

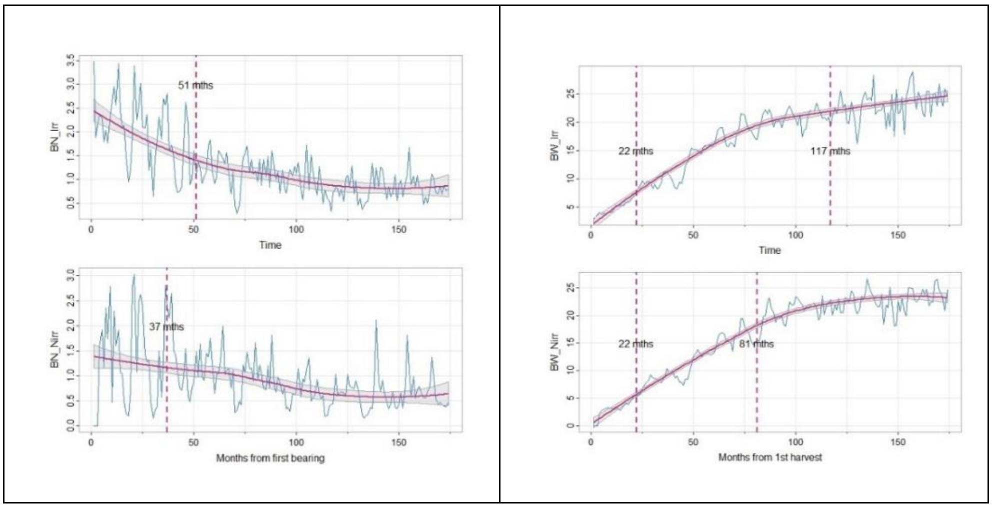


Figure 3 Breakpoints in BN (R) and BW (L) trends for irrigated (irr) and unirrigated (nirr) palms.

irrigated palms. These breakpoints are when the variability in monthly bunch number begins to reduce. The bunch weight trend stabilizes even sooner, after about 2 years of bearing, in both environments and a further change at the plateauing stage. This latter is sooner in unirrigated palms, at seven years after bearing while in irrigated palms BW continues its steep increase till about the $12^{th}$ year of bearing. The trend slope is gentler thereafter in both. No breakpoints were found in the FFB trends of both irrigated and not irrigated palms the inverse relationship between BN and BW negating their respective breakpoints.

## Singular value decomposition and spectral profiling

Screen plots of the singular values of 84 lagged components for $BN_{nirr}$ and $BW_{irr}$, respectively, are shown in Figure 4. We chose a long window length divisible by 12, hence 84 ≈ ~174/2, to capture any long-term periodicity, while recognising the potential major influence of annual rainfall seasonality. The y-axis of singular values is the explained variance for each component. Clearly, the first component accounts for, by far, most of the variance, particularly for $BW_{irr}$, and the first 10 for > 90% of the variance for all traits (Table 2). This is common with most natural time series. Where two components have the same or similar singular values they point to periodic components as discussed below.

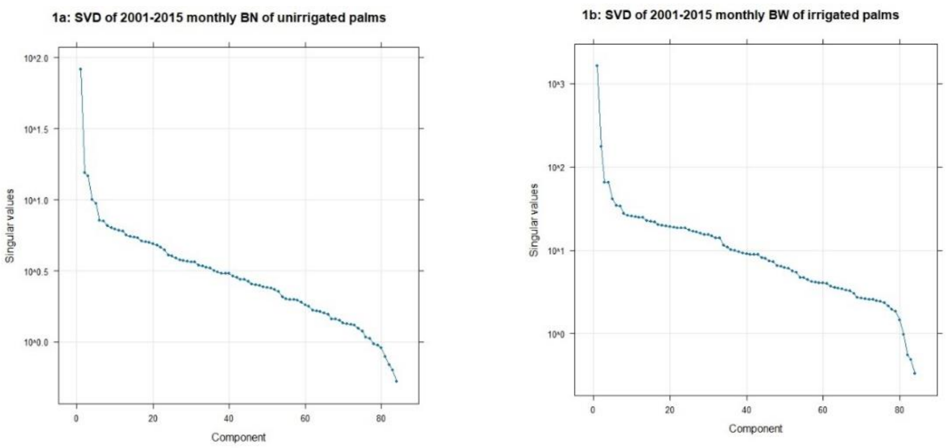


Figure 4 Screen plots of singular values of components from SVD of $BN_{nirr}$ (1a) and $BW_{irr}$ (1b).

Table 2 Proportion (%) of time series variance accounted for by first 10 components from SSA.

| Trait | | Component | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | ∑ |
| Not irrigated | BN | 81.2 | 2.85 | 2.52 | 1.20 | 1.05 | 0.61 | 0.60 | 0.51 | 0.48 | 0.46 | 91.5 |
| | BW | 97.8 | 1.38 | 0.10 | 0.10 | 0.07 | 0.05 | 0.05 | 0.04 | 0.03 | 0.03 | 99.7 |
| | FFB | 80.8 | 1.87 | 1.79 | 1.35 | 1.07 | 1.04 | 0.84 | 0.82 | 0.59 | 0.58 | 90.7 |
| Irrigated | BN | 89.6 | 1.37 | 1.36 | 0.49 | 0.49 | 0.35 | 0.32 | 0.31 | 0.31 | 0.27 | 94.9 |
| | BW | 98.0 | 1.11 | 0.16 | 0.16 | 0.06 | 0.04 | 0.04 | 0.03 | 0.02 | 0.02 | 99.6 |
| | FFB | 88.2 | 1.14 | 1.10 | 0.76 | 0.64 | 0.63 | 0.56 | 0.53 | 0.39 | 0.26 | 94.2 |

Figure 5 shows the patterns of the first 12 eigenvectors of the SVD of $BN_{nirr}$ and $BW_{irr}$, respectively, and their relative contributions to the decomposition. The slowly varying lines denote trends while the more sinusoidal ones suggest periodicity. While the trends are well known in oil palms, the high proportion of variance due to them was still revealing, particularly for BW. In practical terms, the age of palms is the *sine qua non* for estimating BW with one caveat. The BW decomposition shows two trends, albeit with the second accounting for only a small proportion of the variance. The implications of this second trend, as also found in $BW_{nirr}$ decomposition, are discussed below. How well the components are separated, i.e. are not auto correlated, is seen from the weighted correlation matrix (Figure 6) with possible *r*-values from white (*r* =0) to black (*r*=1). The leading single eigentriple(s) describes the exponential trend, while the pairs of subsequent eigentriples correspond to the harmonics, and large 'sparkling' squares indicate white noise. For both $BN_{nirr}$ and $BW_{irr,}$ and, indeed, all traits, the first component, i.e. the trend component, is completely separated from the rest justifying its removal for sequential SSA of the remaining variance. Component 2 of $BW_{irr,}$ which is not a periodic component as seen from the scree plot, is also well separated from the others, except for a correlation (*r*=0.23) with component 5. However, as this value is rather low, this 2nd component was also removed from the repeat SSA of $BW_{irr.}$ In sequential SSA, the

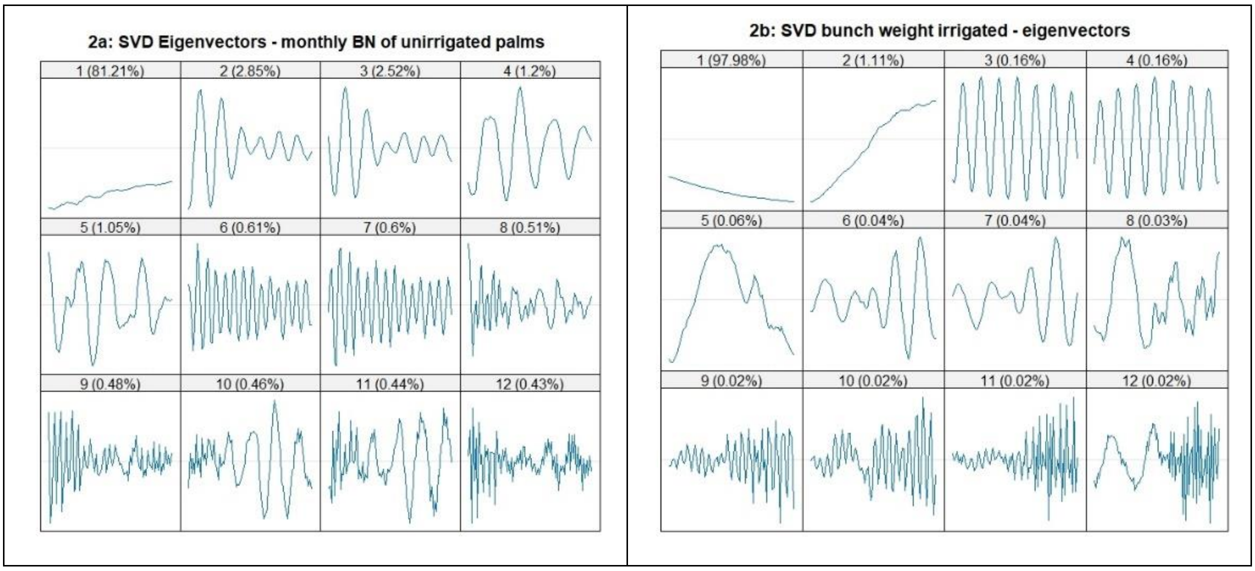


Figure 5 Eigenvector plots of first 12 components of SVD of $BN_{nirr}$ (2a) and $BW_{irr}$ (2b). Values in parenthesis are relative contributions in the decomposition.

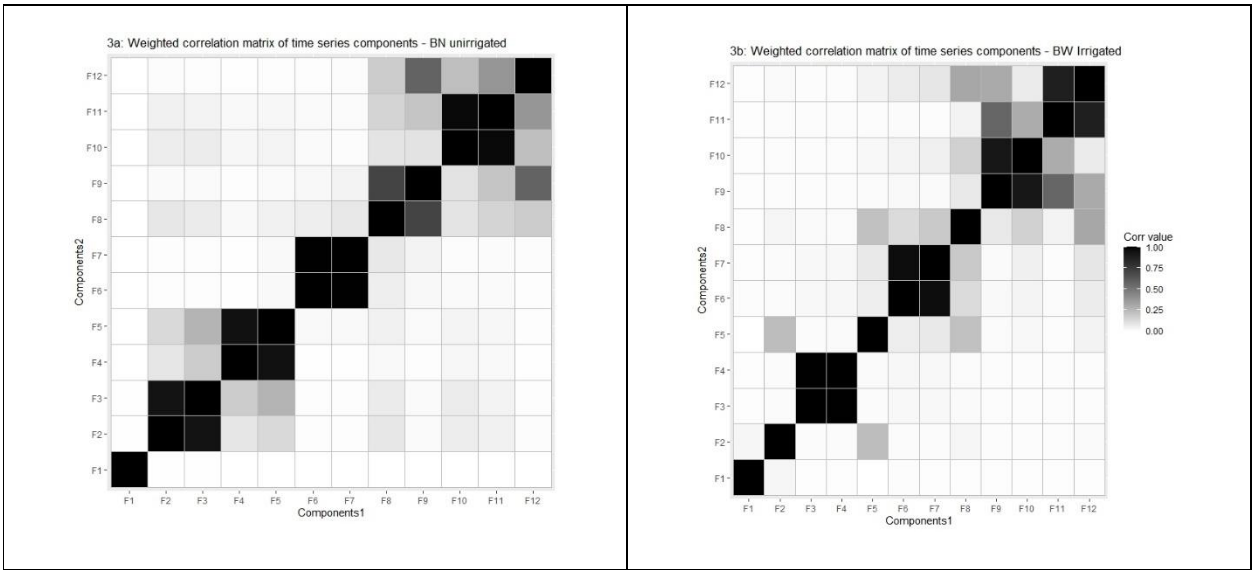


Figure 6 W-correlation matrix of first 12 components of SVD of $BN_{nirr}$ (3a) and $BW_{irr}$ (3b).

residuals, after removing the trend component(s) from the original time series, are decomposed anew to extract the periodic or cyclical components. Figure 7 is the scree plots of the detrended series for $BN_{nirr}$ and $BW_{irr}$, and attention is drawn to the plateaux. A plateau of two components, or eigentriples (ET) of equal value, denotes a periodic series. For $BN_{nirr}$, there is a clear plateau at components 5 and 6 with SVD sigma values 7.2 and 7.1, respectively. There are two other plateaux, albeit less smooth, at components 1 and 2 ($\sigma$=15.4 and 14.6) and at components 3 and 4 ($\sigma$=10.1 and 9.4). For $BW_{irr}$, there are two clear plateaux; at components 1 and 2 and at components 3 and 4. Their sigma values were 65.6 and 65.3 and 34.2 and 33.5, respectively. Pairwise scatterplots of the singular vectors can reveal the eigentriples for harmonic or periodic components (Figure 8). Scatter-plots of pure sine and cosine functions of equal frequencies, amplitudes, and phases are circular. However, if P= 1/w is an integer where w is the frequency, then the scatterplot points are the vertices of a P-vertex polygon. A clear example is the hexagon from the scatterplot of eigentriples 5 vs 6 in Figure 8 (5a); the six vertices of the hexagon denoting a semi-annual periodic component. However, as seen in the next section,

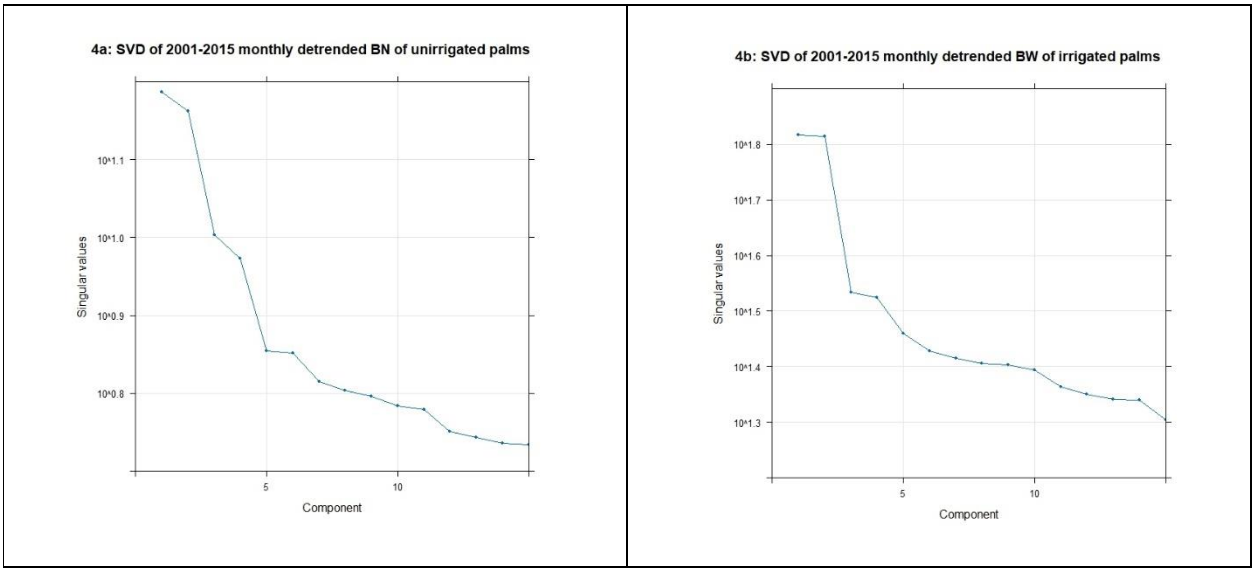


Figure 7 Screen plots of first 15 SVD components of detrended of $BN_{nirr}$ (4a) and $BW_{irr}$ (4b).

not all the periodicities are statistically significant, in which event they are considered residual variance or noise. The Rssa package provides for visualisation of the periodicities by graphing tapered power spectra and their frequences. We compared the output with that produced by the multitaper method (MTM), aided by harmonic F tests for significance, to decide which periodicities are relevant. For harmonic analysis, MTM is considered superior to traditional period-grams and tapered spectral analyses by better balancing bias reduction and variance control. Figure 9 is the power spectrum plot by MTM while Figure 10 gives the F tests of the spectral peaks. The origins of the key peaks and troughs are discussed below. With the isolation of the trend component(s), discovery of periodic or cyclical components and bulking the remaining variance, it is possible to reconstruct the time series. The goodness of reconstruction will indicate the quality of forecasting the time series, but forecasting is not in the ambit of this paper.

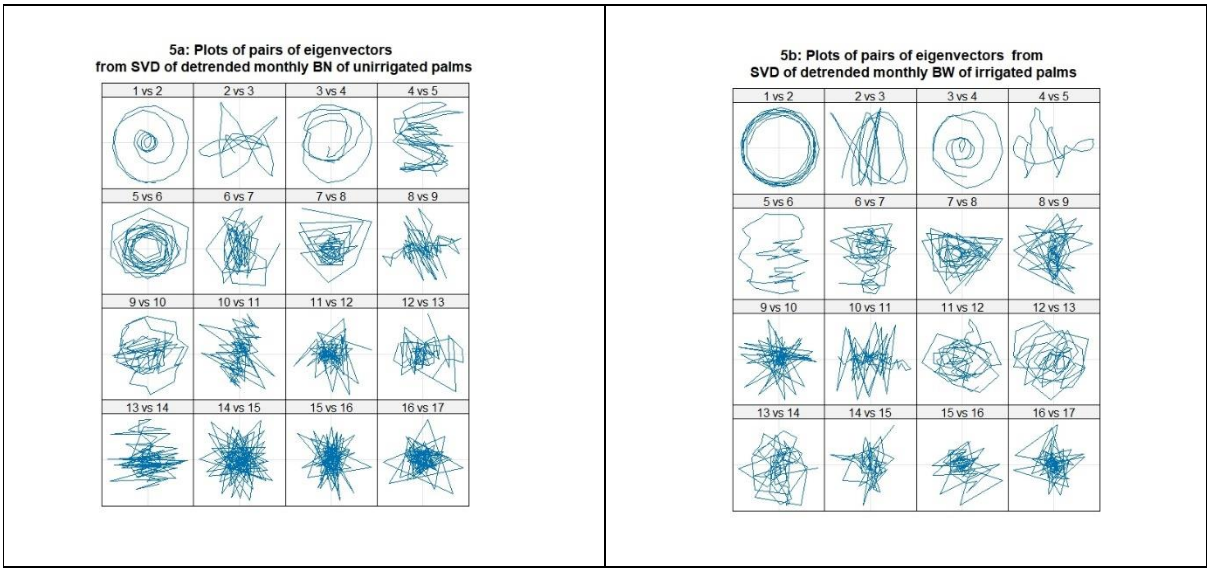


Figure 8 Plots of pairs of eigenvectors of detrended $BN_{nirr}$ (5a) and $BW_{irr}$ (5b).

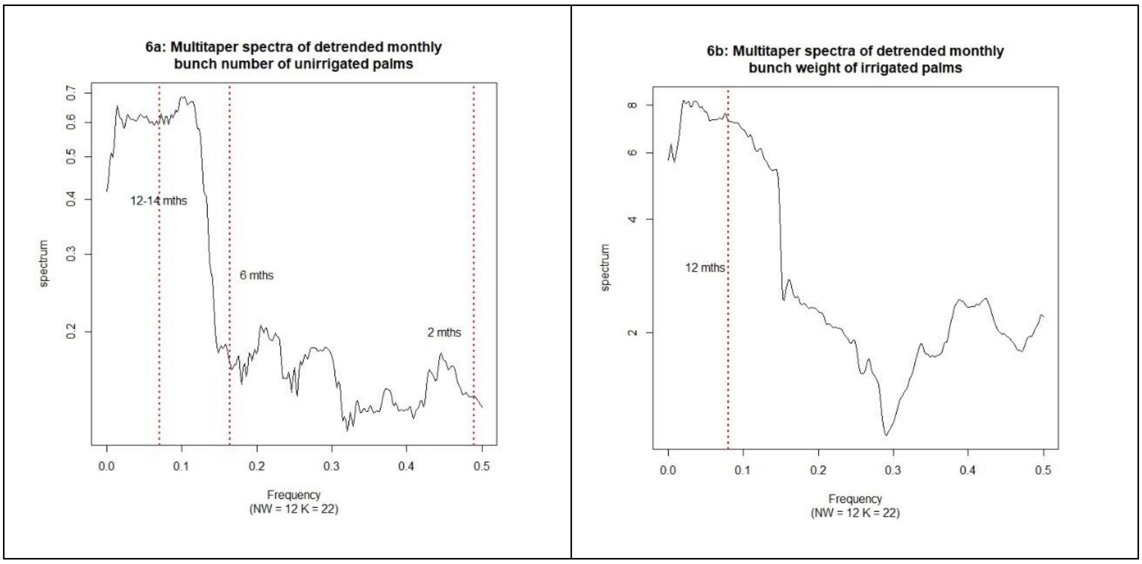


Figure 9 Multitaper spectral plots of detrended of $BN_{nirr}$ (6a) and $BW_{irr}$ (6b).

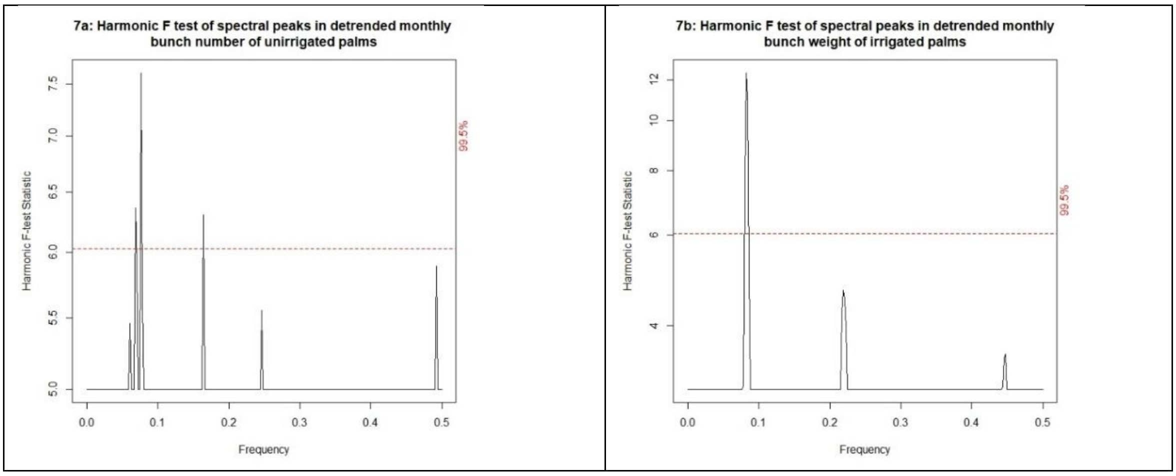


Figure 10 Harmonic F test of MTM spectral peaks of detrended BNnirr (7a) and $BW_{irr}$ (7b). Red dashed line is 99.5% confidence.

Figure 11 shows the reconstructions of BN, BW and FFB time series from 2001–2015 with/without irrigation. The following observations obtain in both irrigated and unirrigated palms BW increases with age while BN declines, these well known in oil palms. Not surprisingly, $BN_{irr}$ was higher than $BN_{nirr}$ at initial bearing but the gap narrowed from about the 4$^{th}$ year of bearing although never quite merging. On the other hand, the gap between the trend lines for $BW_{irr}$ and $BW_{nirr}$ started small at about 2 kg and merged at about the 10$^{th}$ year of bearing at ~22 kg (see also Figure 2). FFB of both irrigated and unirrigated palms declined slightly in the final months but the data are inadequate to support any contention of an age-related decline. Besides the major trend in BW above, a minor, but generally increasing with time, damping was seen for BW in both irrigated and unirrigated palms. The damping declines to the 8$^{th}$/9$^{th}$ year of bearing and then increases. The gross effect is that the bunches started small, then enlarged rapidly before tapering off from the 8$^{th}$/9$^{th}$ year of bearing. This plateau occurred sooner if the hitherto increase was faster, as in good growing conditions with irrigation. BW at this age (10–12 years from planting) is about 19–21 kg. The slowdown in BN also contributed to the plateauing FFB. Oscillating around the trend, are the annual cycles for BN and BW, and, hence also for FFB. 'Annual' is an approximate term as the oscillation durations were from 10–14 months. For BN, the periodicity was more obvious in the young palms, the amplitude (±1) reducing after about the 5$^{th}$ year of bearing. The amplitude was lower in the irrigated palms (±0.5) becoming negligible after about the 9$^{th}$ year of bearing. For $BN_{irr,}$ in the first five years, the peaks were in May/June and the troughs in Nov/Dec shifting, with reducing amplitude, to Oct–Dec and Apr/May, respectively, over the next four years before tapering off. On the other hand, except in the first year, the peaks and troughs in $BN_{nirr}$ were in the same respective months every year. Biologically, we believe the BN annual cycle is a response the annual rainfall cycle with good rains in the 2$^{nd}$ and 3$^{rd}$ quarters and a dry period from Dec–Mar. This prompts the alternating female and male inflorescence production, and the peaks and troughs in BN six months later. The better developed roots of the older palms would provide more access to deeper water, the better to buffers against the vagaries in rainfall and topsoil moisture, hence the diminishing annual cycle with age. The net effect of the annual cycles for BN and BW, particularly for the former, is reflected in similar cycles for FFB. Unlike BN, BW had annual cycles throughout the trial in both the irrigated and unirrigated palms-larger around Jul-Sep and smaller in Dec–Mar. In the irrigated palms the size difference was a regular ±1.5 kg throughout while in the unirrigated palms it was <0.5kg in the initial small bunches to about ±2 kg in the 6–8th year bunch before declining. Of course, as BW increases with age, the monthly fluctuations within a year are more noticeable in young palms. Field observations suggest that the female/ male cycles affecting BN mentioned above, driven by the seasonal rainfall, affects weevil numbers and pollination efficiency and, hence, BW six months later. Besides fewer male inflorescences (weevil breeding sites), weevil activity could also have been lower in the wet months. Poorly pollinated-bunches weigh less.

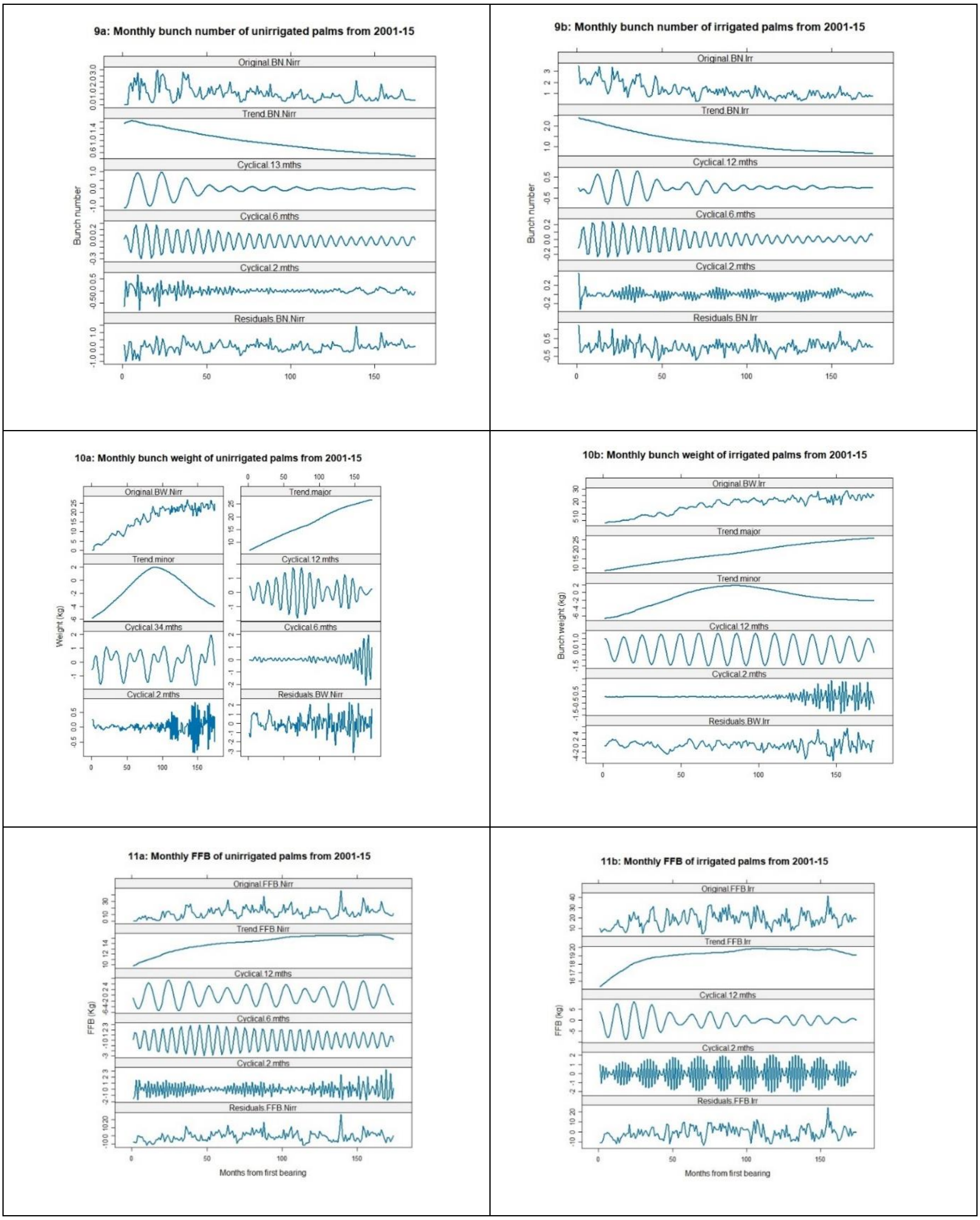


Figure 11 Reconstructed series 2001–2015: monthly BN (top), BW (middle) and FFB (bottom) for unirrigated (L) and irrigated (R) palms.

A semi-annual cycle was detected for BN with peaks in Mar/Apr and Sep/Oct followed by troughs three months later (Figure 12). We suspect that this arose from abortion of developing bunches after a bout of high production. This cycle also decreases with age. While plausible, we did not test whether this was due to more carbohydrate reserves in the older palms. An inexplicable 6-months cycle was also seen in the BW of unirrigated palms with peaks in Dec–Feb and Jun–Aug in the final few years, particularly the last two. The synchrony of this cycle with that of BN resulted in a 6-months cycle in FFB for the unirrigated palms. This cycle was not detected in $BW_{irr}$ and, hence, nor in $FFB_{irr}$. Interestingly, a bimonthly cycle, with peaks in one month and troughs the next, was seen for BN, significant at 99.5%, particularly in the early unirrigated palms, and at 95% confidence for the irrigated throughout the trial. We believe this is due to imprecise harvesting, the under-ripe bunches boosting a month's, followed by a dearth in the next. A similar cycle was detected for BW in the older palms, irrigated or not. This is believed to stem from the artificial bimonthly BN cycle; - BW low in a month without bunch, then high with bunch(es). The oscillation was more pronounced in the older palms with only few large bunches to be harvested. Hence the bimonthly FFB fluctuation can be amplified/diminished by the vagaries of BN and BW; high when both are high, low when both are low and 'average' when one is high and the other low (Figure 13). Lastly, there was a consistent highly significant 34-month cycle in BW of unirrigated palms (Figure 14). We think this arose from the alternative exhaustion and replenishment of carbohydrate reserves every three years. No such cycle occurred in the irrigated palms.

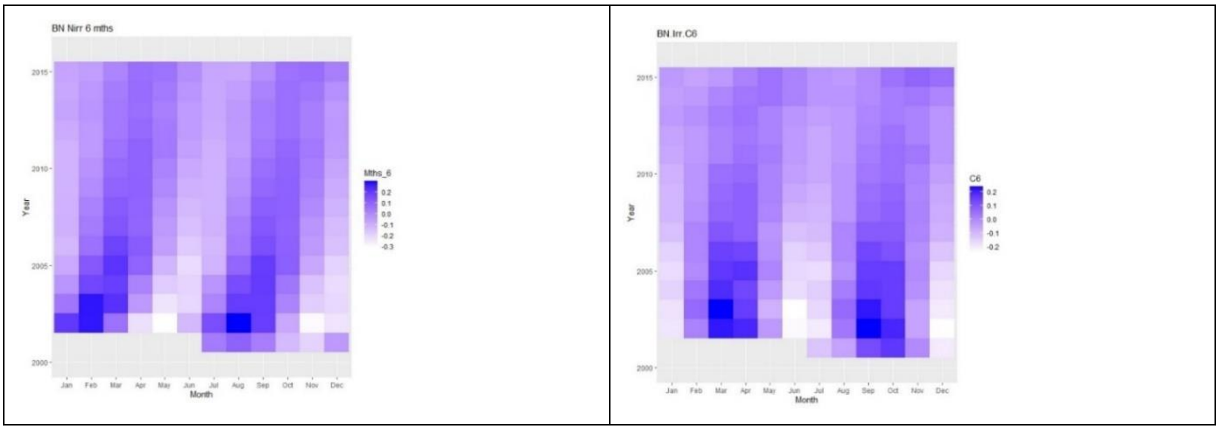


Figure 12 Semi-annual cycles in BN in unirrigated (L) and irrigated (R) palms.

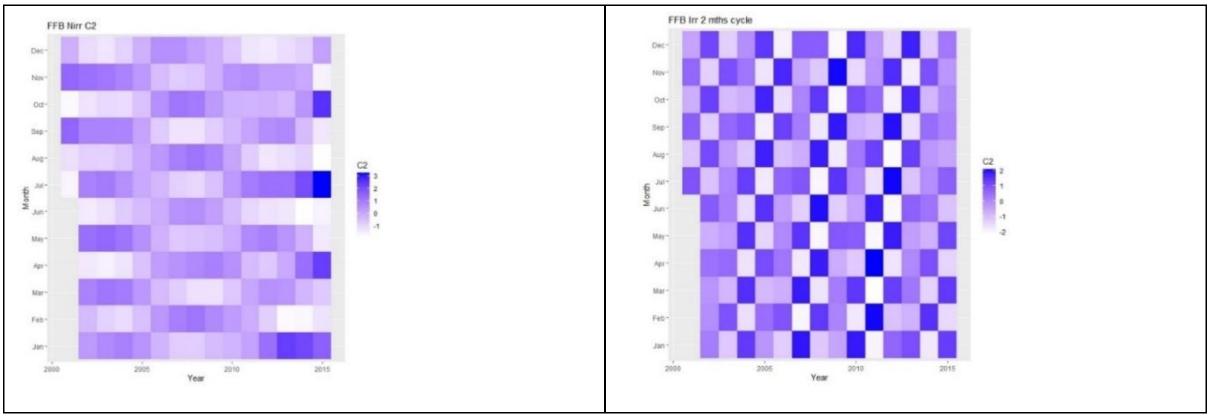


Figure 13 Bimonthly cycles in FFB production in unirrigated (L) and irrigated (R) palms.

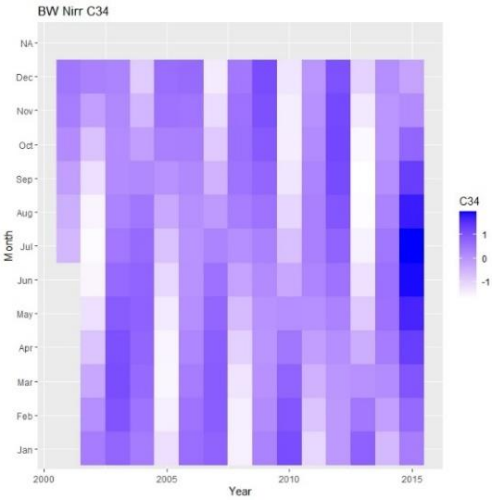


Figure 14 An approximate triennial cycle in BW of unirrigated palms.

## CONCLUSIONS

The main picture of oil palm yield is the trend of fewer BN and increasing BW with age. As the latter outpaces the former, FFB rises. Age accounted for 99% of the variance in BW and 90% and 80% for BN in irrigated and unirrigated palms, respectively. Irrigation gave +38% crop, from +34% BN which averaged +5% larger. While the differences in BN prevailed throughout, BW of irrigated and unirrigated palms converged from about the 7$^{th}$ year of bearing.

The seasonal rains triggered an annual sex cycle of female and male inflorescences, hence high and low BN about six months later. As the palms aged, their more developed roots better buffered them against the vagaries of rainfall and surface soil moisture, and dampened the yield fluctuations, especially for BN. The sex cycle and rain-reduced weevil activity resulted in BW oscillation while harvesting underripe bunches was detected as a bimonthly cycle in BN and, hence, FFB.

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